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AN EFFECTIVE APPROACH TO THE SOLUTION OF TWO-DIMENSIONAL HEAT-CONDUCTION PROBLEMS FOR MULTICONNECTED COMPOSITE BODIES OF COMPLEX SHAPE

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An algorithm to solve two-dimensional nonstationary heat-conduction problems for multiconnected bodies of complex shape, constructed on the basis of potential theory methods with preliminary application of the Rothe method in the time variable, is described. Results of computations are presented for a single- and multilayer strip with holes of arbitrary outline.

It is known that serious calculational difficulties must be encountered in solving boundary-value problems of mathematical physics generally, and of heat conduction, in particular, for domains of complex shape, for example, for those whose boundaries do not agree completely with the coordinate lines of the chosen reference system. Noticeable successes in overcoming these difficulties have been achieved in the construction of calculation algorithms on the basis of variational methods using R-functions to select the coordinate system, finite elements, and summary representations methods [1-3].

For example, the difficulties noted have been overcome sufficiently successfully in [4] in the problem of a homogeneous strip with circular holes. The efficiency of using integral (potential) representation methods for the desired functions [5, 6] is demonstrated below in examples of homogeneous and inhomogeneous strips weakened by holes of arbitrary outline.

§1. Let an infinite strip be weakened by holes arranged periodically over its length. Let us examine part of this strip within the limits of one period and let us formulate the following heat-conduction boundary-value problem for a doubly connected domain Ω (the exterior part of its boundary is the rectangle $0 \le x \le a$, $-b \le y \le b$, and the interior part is an arbitrary closed curve L):

$$\frac{\partial u}{\partial \mathrm{Fo}} = \Delta u, \tag{1.1}$$

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$$u|_{\mathrm{Fo}=0} = \varphi(x, y), \tag{1.2}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=\pm b} = 0, \; A_1 u|_{x=0} = p_1(y, \mathrm{Fo}), \; A_2 u|_{x=a} = p_2(y, \mathrm{Fo}), \tag{1.3}$$

$$Bu|_L = q(x, y, \mathrm{Fo}). \tag{1.4}$$

Here $\mathrm{Fo} = \kappa t/a^2$ is the Fourier criterion, A_1, A_2, and B are linear differential operators of not higher than first order, and p_1, p_2, and q are known functions of their arguments.

Applying the idea of the method of lines with the derivative with respect to Fo replaced by a finite-difference relation on the plane $\mathrm{Fo}_{k+1} = (k + 1)t$ ($k = 0, 1, 2, \ldots$; t is the spacing between two successive sections Fo = const), we obtain a boundary-value problem for the functions $u_{k+1} = u(x, y, \mathrm{Fo}_{k+1})$

$$\Delta u_{k+1} - \frac{1}{t} u_{k+1} = -\frac{1}{t} u_k \text{ in } \Omega, \tag{1.5}$$

$$A_1 u_{k+1}|_{x=0} = p_1^{k+1}(y), \quad A_2 u_{k+1}|_{x=a} = p_2^{k+1}(y), \tag{1.6}$$

$$\left.\frac{\partial u_{k+1}}{\partial y}\right|_{y=\pm b} = 0, \; Bu_{k+1}|_L = q^{k+1}(x, y). \tag{1.7}$$

Without limiting the generality of the discussion, we can evidently examine the problem

$$(\Delta - \nu) u = F(x, y), \tag{1.8}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=\pm b} = 0, \; A_1 u|_{x=0} = 0, \; A_2 u|_{x=a} = 0, \tag{1.9}$$

$$Bu|_L = Q(x, y) \tag{1.10}$$

in place of (1.5)-(1.7).

A Green's function of the problem (1.8)-(1.9) can easily be constructed for the rectangle $0 \le x \le a$, $-b \le y \le b$, as can be done by separation of variables and subsequent application of the method of variation of arbitrary constants, for example. Thus, for $A_1 \equiv 1$, $A_2 \equiv d/dx + \alpha$, $a = 1$ it can be represented by such a method by

$$g(x, y; \xi, \eta) = \sum_{k=0}^{\infty} g_k(x, \xi) \cos k\pi y/b \cos k\pi\eta/b, \tag{1.11}$$

where

$$g_0(x, \xi) = \begin{cases} \dfrac{\alpha x - \alpha - 1}{\alpha + 1}\,\xi, & x \geqslant \xi, \\ \dfrac{\alpha\xi - \alpha - 1}{\alpha + 1}\,x, & x \leqslant \xi, \end{cases}$$

$$g_k(x, \xi) = \begin{cases} \left(\dfrac{\nu \operatorname{sh} \nu + \alpha \operatorname{ch} \nu}{\nu \operatorname{ch} \nu + \alpha \operatorname{sh} \nu} \operatorname{sh} \nu x - \operatorname{ch} \nu x\right) \dfrac{\operatorname{sh} \nu\xi}{\nu}, & x \geqslant \xi, \\ \left(\dfrac{\nu \operatorname{sh} \nu + \alpha \operatorname{ch} \nu}{\nu \operatorname{ch} \nu + \alpha \operatorname{sh} \nu} \operatorname{sh} \nu\xi - \operatorname{ch} \nu\xi\right) \dfrac{\operatorname{sh} \nu x}{\nu}, & x \leqslant \xi, \; (k = 1, 2, \ldots). \end{cases}$$

Then the solution of the problem (1.8)-(1.10) can be written as the sum

$$u(x, y) = v(x, y) + w(x, y),$$

where v(x, y) is the solution of the problem (1.8)-(1.9) representable by the known Hilbert theory ([7], p. 177) by the formula

$$v(x, y) = \iint_{\Omega} g(x, y; \xi, \eta) F(\xi, \eta) d_{\xi,\eta} \Omega; \tag{1.12}$$

w(x, y) is a function which we shall seek in the form

$$w(x, y) = \int_L g(x, y; \xi, \eta) \mu(\xi, \eta) d_{\xi,\eta} L.$$

The weight (density) $\mu(\xi, \eta)$ of this latter representation is determined to satisfy the condition (1.10) from an equation of the form

$$R(x, y) = \int_L T(x, y; \xi, \eta) \mu(\xi, \eta) d_{\xi,\eta} L. \tag{1.13}$$

Here

$$R(x, y) = Q(x, y) - Bv(x, y)|_L,$$
$$T(x, y; \xi, \eta) = Bg(x, y; \xi, \eta).$$

As later computations show, obtaining the approximate solution of (1.13) is not associated with essential calculation difficulties. It is obtained with sufficient accuracy below by the method of quadrature formulas.

The stationary temperature fields of a strip weakened by elliptical and trapezoidal holes with the initial data $A_1 \equiv B \equiv 1$, $A_2 = d/dx + \alpha$, $p_1 \equiv q \equiv 1$, $p_2 \equiv 0$, $\alpha = 1$ are shown in Fig. 1a. If we set $\alpha = 0$, then the boundary condition obtained will correspond to an adiabatic edge $x = a$ of the strip and, therefore, the boundary-value problem for a strip weakened by two series of holes (Fig. 2) will be solved in this case. Here $p_1 \equiv 0$.

The error in the approximate calculation of the integral (1.12) is easily checked and w(x, y) is a harmonic function by construction; hence, by virtue of the maximum principle the error of the solution to the problem (1.8)-(1.10) can be judged by the accuracy of satisfying the boundary condition on the contour L, since the boundary conditions are satisfied identically on the outer part of the boundary because of the governing properties of the Green's function. The error mentioned was a maximum at the points A and did not exceed 0.4% for the cases considered [with 18 quadrature nodes when replacing the integral in (1.13) by a finite sum].

The results of computing the temperature field of a strip for the case of discontinuous boundary conditions on the contour L

$$u|_{\widehat{BC}} = 1, \ u|_{CDEB} = 0$$

are represented in Fig. 1b.

The solution for other domains Ω with a more complex outer boundary can evidently also be obtained by using the algorithm described if the construction of their Green's function is possible (a strip, half-strip, circle, circular sector, ring), where the shape of the contour L imposes no substantial constraints on the applicability of this method.

§2. The algorithm elucidated above for the solution of boundary-value problems of heat conduction for a single-layer strip is extended in a natural way to the case of multilayer strips. Let it be required to find the solution of the stationary heat-conduction boundary-value problem for an n-layered strip:

$$\Delta u_i = f_i \quad (i = \overline{1, n}), \tag{2.1}$$

$$A_1 u_1|_{x=a_0} = 0, \tag{2.2}$$

$$u_i|_{x=a_i} = u_{i+1}|_{x=a_i}, \ \lambda_i \left.\frac{\partial u_i}{\partial x}\right|_{x=a_i} = \lambda_{i+1} \left.\frac{\partial u_{i+1}}{\partial x}\right|_{x=a_i} \quad (i = \overline{1, n-1}), \tag{2.3}$$

$$A_2 u_n|_{x=a_n} = 0, \tag{2.4}$$

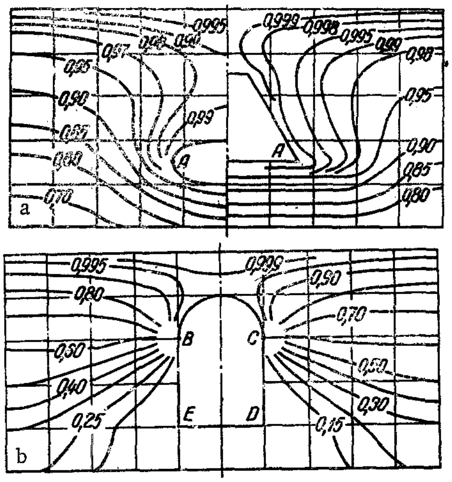

Fig. 1. Dimensionless temperature distribution in a strip weakened by holes of a different shape.

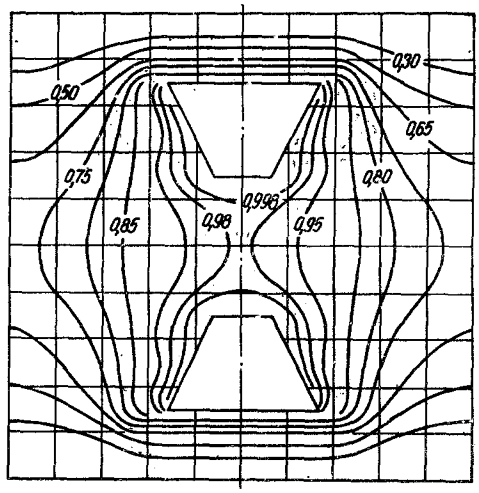

Fig. 2. Dimensionless temperature field in a strip with two series of trapezoidal holes.

$$\left.\frac{\partial u_i}{\partial y}\right|_{y=\pm b} = 0 \quad (i = \overline{1, n}), \tag{2.5}$$

$$Bu_i|_L = Q. \tag{2.6}$$

If the Green's matrix of the problem (2.1)-(2.5) is known, then the solution of the boundary-value problem (2.1)-(2.6) can be obtained by analogy with the previous problem.

Let us mention one of the possible algorithms for constructing the Green's matrix of the boundary-value problem (2.1)-(2.5), namely, let us assume representability of the functions u_i, f_i by the expansions

$$u_i = \sum_{n=0}^{\infty} u_{i,n}(x) \cos n\pi y/b,$$
$$f_i = \sum_{n=0}^{\infty} f_{i,n}(x) \cos n\pi y/b. \tag{2.7}$$

This will result in the following boundary-value problem for a system of ordinary differential equations:

$$u''_{i,n}(x) - \nu^2 u_{i,n}(x) = f_{i,n} \quad (i = \overline{1, n}), \tag{2.8}$$

$$A_1 u_{1,n}|_{x=a_0} = 0, \tag{2.9}$$

$$u_{i,n}|_{x=a_i} = u_{i+1,n}|_{x=a_i}; \; \lambda_i u'_{i,n}|_{x=a_i} = \lambda_{i+1} u'_{i+1,n}|_{x=a_i} \quad (i = \overline{1, n-1}), \tag{2.10}$$

$$A_2 u_{n,n}|_{x=a_n} = 0. \tag{2.11}$$

Here

$$\nu = n\pi/b.$$

We construct the Green's matrix of this last problem by applying the Lagrange method of variation of arbitrary constants. Let us write the algorithm of its construction for a two-layered strip ($-a_2 \le x \le a_1$). As is known, for n > 0 the general solution of the homogeneous equations corresponding to (2.8) can be written as follows:

$$u_{i,n} = c_{i1}(x) \operatorname{sh} \nu x + c_{i2} \operatorname{ch} \nu x \quad (i = 1,2).$$

The systems of linear algebraic equations of the Lagrange method will appear as follows:

$$c_{i1}'(x)\,\mathrm{sh}\,\nu x + c_{i2}'(x)\,\mathrm{ch}\,\nu x = 0$$
$$c_{i1}'(x)\,\nu\,\mathrm{ch}\,\nu x + c_{i2}'(x)\,\nu\,\mathrm{sh}\,\nu x = f_{i,n}\cdot \qquad (i = 1,\ 2).$$

Hence,

$$c_{i1}' = \frac{f_{i,n}\,\mathrm{ch}\,\nu x}{\nu},\quad c_{i2}' = -\frac{f_{i,n}\,\mathrm{sh}\,\nu x}{\nu} \quad (i = 1,\ 2).$$

The general solution of (2.8) is hence represented by the formulas

$$u_{1,n} = \int_{-a_2}^{0} \omega_1(x,\ \xi) f_{1,n}(\xi)\,d\xi + c_{11}^n\,\mathrm{sh}\,\nu x + c_{12}^n\,\mathrm{ch}\,\nu x,$$
$$u_{2,n} = \int_{0}^{a_1} \omega_2(x,\ \xi) f_{2,n}(\xi)\,d\xi + c_{21}^n\,\mathrm{sh}\,\nu x + c_{22}^n\,\mathrm{ch}\,\nu x,$$

where

$$\omega_1(x,\ \xi) = \begin{cases} \dfrac{\mathrm{sh}\,\nu(x-\xi)}{\nu}, & -a_2 \leqslant \xi \leqslant x \leqslant 0, \\ 0 & -a_2 \leqslant x \leqslant \xi \leqslant 0, \end{cases}$$
$$\omega_2(x,\ \xi) = \begin{cases} \dfrac{\mathrm{sh}\,\nu(x-\xi)}{\nu}, & 0 \leqslant \xi \leqslant x \leqslant a_1, \\ 0 & 0 \leqslant x \leqslant \xi \leqslant a_1. \end{cases}$$

The constants c_{ij}^n (i, j = 1, 2) are determined by complying with the boundary conditions (2.9)-(2.11). For $A_1 \equiv 1$, $A_2 \equiv d/dx + \alpha$, $\lambda = \lambda_1/\lambda_2$ the elements of the Green's matrix $g^n(x, \xi) = (g_{ij}^n(x,\ \xi))_{ij=1,2}$ are determined by the expressions

$$g_{11}^n = \begin{cases} \dfrac{k_2(a_1,\ 0)\,\mathrm{sh}\,\nu x - \lambda k_1(a_1,\ 0)\,\mathrm{ch}\,\nu x}{D}\,k_3(1,\ \xi), & -a_2 \leqslant \xi \leqslant x \leqslant 0, \\ \dfrac{k_2(a_1,\ 0)\,\mathrm{sh}\,\nu\xi - \lambda k_1(a_1,\ 0)\,\mathrm{ch}\,\nu\xi}{D}\,k_3(1,\ x), & -a_2 \leqslant x \leqslant \xi \leqslant 0, \end{cases}$$

$$g_{12}^n = -\frac{k_1(a_1,\ \xi)}{D}\,k_3(1,\ x),\quad -a_2 \leqslant x \leqslant 0,\ 0 \leqslant \xi \leqslant a_1,$$

$$g_{21}^n = \frac{k_2(a_1,\ 0)\,\mathrm{sh}\,\nu x - k_1(a_1,\ 0)\,\mathrm{ch}\,\nu x}{D}\,\lambda k_3(1,\ \xi),\quad \begin{matrix} 0 \leqslant x \leqslant a_1, \\ -a_2 \leqslant \xi \leqslant 0, \end{matrix}$$

$$g_{22}^n = \begin{cases} -\dfrac{k_1(a_1,\ x)}{D}\,k_3(\lambda,\ \xi), & 0 \leqslant \xi \leqslant x \leqslant a_1, \\ -\dfrac{k_1(a_1,\ \xi)}{D}\,k_3(\lambda,\ x), & 0 \leqslant x \leqslant \xi \leqslant a_1, \end{cases}$$

and the parameter D by the relationship

$$D = \nu\,[\lambda k_1(a_1,\ 0) + \mathrm{th}\,\nu a_2 k_2(a_1,\ 0)],$$

where

$$k_1(x,\ \xi) = \nu\,\mathrm{ch}\,\nu(x-\xi) + \alpha\,\mathrm{sh}\,\nu(x-\xi),$$
$$k_2(x,\ \xi) = \nu\,\mathrm{sh}\,\nu(x-\xi) + \alpha\,\mathrm{ch}\,\nu(x-\xi),\quad k_3(\lambda,\ x) = \lambda\,\mathrm{sh}\,\nu x + \mathrm{th}\,\nu a_2\,\mathrm{ch}\,\nu x.$$

The case n = 0 in (2.8) requires an independent approach and we easily obtain

$$g_{11}^0 = \begin{cases} x - \xi - k_4(1,\ x)\,k_5(\lambda,\ \xi)/T, & -a_2 \leqslant \xi \leqslant x \leqslant 0, \\ -k_4(1,\ x)\,k_5(\lambda,\ \xi)/T, & -a_2 \leqslant x \leqslant \xi \leqslant 0, \end{cases}$$

$$g_{12}^0 = -k_4(1, x)k_5(1, \xi)/T, \quad -a_2 \leqslant x \leqslant 0, \ 0 \leqslant \xi \leqslant a_1,$$

$$g_{21}^0 = -\lambda k_4(1, \xi)k_5(1, x)/T, \quad -a_2 \leqslant \xi \leqslant 0, \ 0 \leqslant x \leqslant a_1,$$

$$g_{22}^0 = \begin{cases} x - \xi - k_4(\lambda, x)k_5(1, \xi)/T, & 0 \leqslant \xi \leqslant x \leqslant a_1, \\ -k_4(\lambda, x)k_5(1, \xi)/T, & 0 \leqslant x \leqslant \xi \leqslant a_1. \end{cases}$$

Here

$$T = \lambda + \alpha a_2 + \alpha\lambda a_1,$$

$$k_4(\lambda, x) = \lambda x + a_2, \ k_5(\lambda, x) = \lambda + \alpha\lambda a_1 - \alpha x.$$

Now, having the Green's matrix of the problem (2.8)-(2.11) let us use the already-mentioned Hilbert theorem about partial solutions of inhomogeneous systems of ordinary differential equations. After this, we apply the Fourier—Euler formula for the coefficients of the second of the expansions (2.7), which finally permits the elements $g_{ij}(x, y; \xi, \eta)$ of the Green's matrix for the problem (2.1)-(2.5) to be determined by the formula

$$g_{ij}(x, y; \xi, \eta) = \frac{2}{b}\sum_{n=0}^{\infty} \frac{1 + \operatorname{sign} n}{2} g_{ij}^n(x, \xi)\cos \nu y \cos \nu\eta.$$

The solution of the boundary-value problem for a two-layered strip weakened by holes or grooves can be obtained by using the construction of the Green's matrix analogously to the solution of the problem (1.8)-(1.10) by using the Green's function (1.11) performed above.

The stationary temperature field of a two-layered strip weakened by an elliptical hole ($x = 0.2 \cos\varphi$; $y = 0.6 \sin\varphi$) for the following values of the operators, functions, and constants in the formula $A_1 \equiv B \equiv 1$, $A_2 \equiv d/dx + 1$, $f_1 \equiv f_2 \equiv 1$: $a_1 = 0.5$, $a_2 = 0.5$, $b = 1$, $Q = \cos(\varphi/2)$, $\lambda = 100$ (such a ratio corresponds to a steel—aluminum pair, for example) is shown by level lines in Fig. 3a.

The temperature field originating in a two-layered strip with a circular groove whose contour intersects the interface between the layers is presented in Fig. 3b. Here the initial data were: $A_1 \equiv B \equiv 1$, $A_2 \equiv d/dx + 1$, $f_1 \equiv 1$, $f_2 \equiv 1$, $a_1 = 0.7$, $a_2 = 0.3$, $b = 1$, $Q \equiv 0$, $\lambda = 100$.

The error in complying with the boundary conditions on the hole outline did not exceed 1% in the cases mentioned.

Therefore, the results presented in this paper permit making a conclusion about the high efficiency of the illustrated method of potential representation of the desired quantities in constructing calculation algorithms for the solution of heat-conduction boundary-value problems for multiconnected inhomogeneous domains of complex shape.

In conclusion, let us mention that the results used in this paper for the numerical realization of the algorithm described were obtained by using a TA-1M translator with the algorithmic language ALGOL-60 on an "M-222" electronic digital computer. The time to solve the problem was hence about 10 min.

NOTATION

Ω, domain under consideration; x, y, ξ, η, coordinates; a, width of the strip; b, half the spacing between the centers of the holes; L, hole boundary; u, temperature; $g(x,y; \xi,\eta)$,

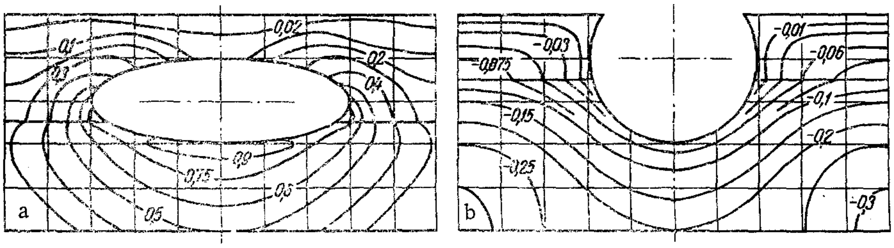

Fig. 3. Dimensionless temperature distribution in two-layered strips ($\lambda = \lambda_1/\lambda_2 = 100$).

Green's function; $g_{ij}(x,y;\ \xi,\eta)$, elements of the Green's matrix; $\mu(x,y)$, density; λ_i, thermal conductivity coefficients.

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DETERMINATION OF THE GEOMETRIC-OPTICS COEFFICIENTS OF THERMAL RADIATION BY THE MONTE CARLO METHOD

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UDC 536.3

Algorithms of the Monte Carlo method to determine the governing angular coefficients for different formulations of the radiant exchange problem under conditions of a diathermal medium and results of their verification by means of exact solutions are presented.

The method of statistical tests, or the Monte Carlo method [1-6], has recently been applied quite frequently to the solution of applied radiant heat-transfer problems. In the case of systems filled with a diathermal medium, this method is used principally for the direct determination of the geometric-optics characteristics of the radiation field [2-5]. Let us examine the question of applying the Monte Carlo method to obtain directly one such characteristic, the governing angular coefficient [7, 8]. Let us take the usual assumptions about the diffuseness of the radiation and the grayness and opacity of the system boundaries. Let us limit ourselves to finding the mean value of the coefficient of greatest interest in engineering practice. Let us assume that the system under investigation consists of a finite number of zones (bodies), within each of whose limits the given optical and energetic characteristics are constant from point to point.

The possibility of a statistical modeling of the governing angular coefficient is based on its representation as an infinite functional series [7] (whose first member is the geometric angular coefficient, and the next terms of the series take into account the first, second, and all the remaining reflections) which expresses the method of multiple reflections explicitly. Therefore, the desired coefficient can be determined by observing the fate of the different rays in time. The probabilistic treatment of the mean governing angular coefficient Φ_{ik} as a characteristic of the fraction of proper radiation of the zone i reaching the zone k directly and taking into account all the re-reflections in the system is also used in constructing the algorithm of the Monte Carlo method.

The field of governing angular coefficients is ordinarily found from the solution of integral equations of the resolvent of the initial integral equations of radiation transfer.

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